

Isothermal Gravitational Segregation with Damping

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At constant temperature, the equilibrium variations in composition and pressure due to gravitational segregation within an oil or gas reservoir are given by the equations

$$RT\Delta \ln z_i = -RT\Delta \ln(\phi_i p) - M_i g \Delta h, \quad (1)$$

$$\ln \left(\sum_i z_i \right) = 0, \quad (2)$$

where z_i is the mole fraction of component i , p is the pressure, T is the temperature, h is the height above an arbitrary reference point, ϕ_i is the fugacity coefficient of component i (as a function of temperature, pressure and composition), M_i is the molecular weight of component i , g is the gravitational acceleration constant, and R is the universal gas constant. The initial pressure (p°) and mole fractions (z_i°), corresponding to $\Delta h = 0$, must be given quantities.

In some cases, for whatever reason, the fluids within a petroleum reservoir may not reach a gravitational/chemical equilibrium. In such cases, we would still like to have a model of the composition and pressure variations. At one extreme from the model of perfect equilibrium is the model of perfect mixing, in which the composition is constant everywhere (except across a phase-change boundary, such as a gas-oil contact) and the pressure differential at each point is given by

$$dp = -\rho g dh, \quad (3)$$

where ρ is the fluid density at the given temperature, pressure and composition.

Early in 2003, Zick Technologies developed a new model for isothermal gravitational segregation that encompasses the entire spectrum from perfect equilibrium to perfect mixing. The new model is given by the equations

$$RTw_i = -RT\Delta \ln(\phi_i p) - M_i g \Delta h, \quad (4)$$

$$\frac{1}{\alpha} \ln \left(\sum_i z_i^\circ e^{\alpha w_i} \right) = 0, \quad (5)$$

$$\Delta \ln z_i = \alpha w_i, \quad (6)$$

where α is a *damping* factor that can be assigned any value from 0 to 1. In addition, w_i is a specially chosen intermediate variable for component i that is roughly proportional to Δh but relatively independent of α . When $\alpha = 1$, the new model reverts to equations (1)–(2) and the compositional changes are determined by thermodynamic equilibrium. When $\alpha = 0$, the composition is held constant, as shown by equation (6). For intermediate values of α , the compositional variation will be somewhere between the two extremes. Regardless of α , it can be shown that the pressure variation will always maintain the hydrostatic equilibrium given by equation (3).

When $\alpha = 1$, the changes in composition and pressure can be determined, reversibly, for any change in height, even across phase-change boundaries. In other words, there are no step-size dependencies and any series of height changes that ultimately returns the height to its original value will also return the composition and pressure to their original values, even if phase-change boundaries have been crossed along the way.

For $\alpha < 1$, the composition and pressure changes are reversible only if the fluid remains single-phase from one height to the next (not counting the discontinuous transitions at phase-change boundaries). The fluid always remains single-phase for $\alpha = 1$, but not necessarily for $\alpha < 1$. For example, suppose an oil is single-phase at a certain reference height and suppose $\alpha = 0$. As the height is increased, the pressure will decrease, but the saturation pressure remains the same because of the constant composition. Eventually, at the gas-oil contact height, the pressure will equal the saturation pressure and the fluid will change from oil to gas. However, any further increase in height will again cause the pressure to decline, now taking it below the constant dew-point pressure of the gas, causing some retrograde condensation. If α had equaled 1, on the other hand, the composition of the gas would have changed rapidly enough for the dew-point pressure to decrease faster than the pressure, keeping the gas single-phase. In fact, there will be some critical value of α , above which the gas will remain single-phase as the height is increased and below which the gas will undergo a phase split.

It is not really appropriate to apply the new model with a constant value of α that causes multiple phases to appear at any of the heights of interest (except at the discontinuities of the phase-change boundaries). Such cases would violate the model's assumption that the pressure changes are determined by the integration of single-phase densities over the changes in height. In fact, because different phases have different densities, multiple phases should actually have separate pressures (except at the phase-change boundaries). The difference in pressure between two separate phases in contact with each other is the so-called *capillary* pressure. However, no current thermodynamic model, including the one presented here, is known to handle equilibrium between phases at different pressures (i.e., to account properly for this capillary pressure effect). Should the new model predict the appearance of multiple phases above (or below) a certain height, then a different, larger value of α should be

applied, at least above (or below) that height (if not universally), to keep the fluids single-phase (except at the inevitable phase-change boundaries).

Regardless of α , all gravitational segregation calculations must originate from a given reference point with a specified composition and pressure. From that point, upward integration should be initialized with the least dense of all equilibrium phases present, while downward integration should be initialized with the densest phase. If a phase-change boundary is encountered during the integration, then any further upward integration from that point should be reinitialized with the least dense of the equilibrium phases at that point, while further downward integration should be reinitialized with the densest phase. If further integration (from a phase-change boundary) continues to produce multiple phases, then α should be increased to keep the fluid single-phase and the model valid.